

تاریخ

EE362

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Q. Find the transfer function $G(s) = \frac{C(s)}{R(s)}$, corresponding to the differential equation:

$$\frac{d^3 c}{dt^3} + 3 \frac{d^2 c}{dt^2} + 7 \frac{dc}{dt} + 5c = \frac{d^2 r}{dt^2} + 4 \frac{dr}{dt} + 3r$$

By taking the Laplace transform of the differential equation assuming zero initial

$$\star \mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0)$$

$$\star \mathcal{L}\left[\frac{d^2 f}{dt^2}\right] = s^2 F(s) - sf(0) - \dot{f}(0)$$

$$\star \mathcal{L}\left[\frac{d^{\tilde{n}} f}{dt^{\tilde{n}}}\right] = s^{\tilde{n}} F(s) - \sum_{k=1}^{\tilde{n}} s^{\tilde{n}-k} f^{(k-1)}(0)$$

$$s^3 C(s) + 3s^2 C(s) + 7sC(s) + 5C(s) = s^2 R(s) + 4sR(s) + 3R(s)$$

$$C(s)(s^3 + 3s^2 + 7s + 5) = R(s)(s^2 + 4s + 3)$$

$$\text{T.F } G(s) = \frac{C(s)}{R(s)} = \frac{s^2 + 4s + 3}{s^3 + 3s^2 + 7s + 5}$$

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Q. Find the differential equation corresponding to the transfer function:-

$$G(s) = \frac{2s+1}{s^2+6s+2}$$

$$G(s) = \frac{C(s)}{R(s)} = \frac{2s+1}{s^2+6s+2} \implies R(s)(2s+1) = C(s)(s^2+6s+2)$$

By taking inverse Laplace transform assuming zero initial conditions,

$$\frac{d^2c}{dt^2} + 6\frac{dc}{dt} + 2c = 2\frac{dr}{dt} + r$$

Q. Find the ramp response for a system whose transfer function is

$$G(s) = \frac{s}{(s+4)(s+8)}$$

$$G(s) = \frac{C(s)}{R(s)} \implies C(s) = R(s)G(s) \quad \left| \text{ramp}(t) = \frac{1}{s^2} \right|$$

$$C(s) = \frac{1}{s^2} * \frac{s}{(s+4)(s+8)} = \frac{1}{s(s+4)(s+8)} \quad | 2 |$$

$$C(s) = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+8}$$

$$A = \frac{1}{(s+4)(s+8)} \Big|_{s=0} \Rightarrow \boxed{A = \frac{1}{32}}, \quad B = \frac{1}{s(s+8)} \Big|_{s=-4}$$

$$\Rightarrow \boxed{B = -\frac{1}{16}}, \quad C = \frac{1}{s(s+4)} \Big|_{s=-8} \Rightarrow \boxed{C = \frac{1}{32}}$$

$$C(s) = \frac{1}{32s} + \frac{1}{32(s+8)} - \frac{1}{16(s+4)}$$

By taking inverse Laplace transform

$$C(t) = \frac{1}{32} + \frac{1}{32} e^{-8t} - \frac{1}{16} e^{-4t} \quad \neq$$

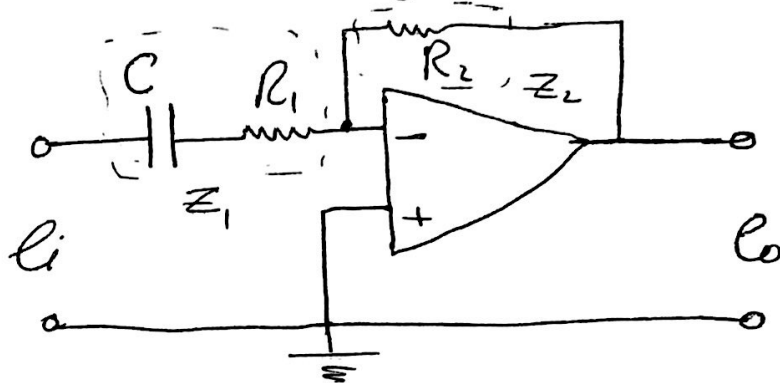
$$\begin{aligned} \delta(t) &= 1 \\ u(t) &= \frac{1}{s} \\ t u(t) &= \frac{1}{s^2} \\ t^n u(t) &= \frac{n!}{s^{n+1}} \\ e^{\lambda t} u(t) &= \frac{1}{s-\lambda} \\ e^{\lambda t} t u(t) &= \frac{1}{(s-\lambda)^2} \\ e^{\lambda t} t^n u(t) &= \frac{n!}{(s-\lambda)^{n+1}} \end{aligned}$$

$$\begin{aligned} \cos bt \cdot u(t) &= \frac{s}{s^2+b^2} \\ \sin bt \cdot u(t) &= \frac{b}{s^2+b^2} \\ e^{-at} \cos bt \cdot u(t) &= \frac{s+a}{(s+a)^2+b^2} \\ e^{-at} \sin bt \cdot u(t) &= \frac{b}{(s+a)^2+b^2} \end{aligned}$$

$$\boxed{u(t)=1}$$

$$\boxed{3}$$

find the transfer function $\frac{E_o(s)}{E_i(s)}$ of the op-amp *
circuit shown in figure below:-



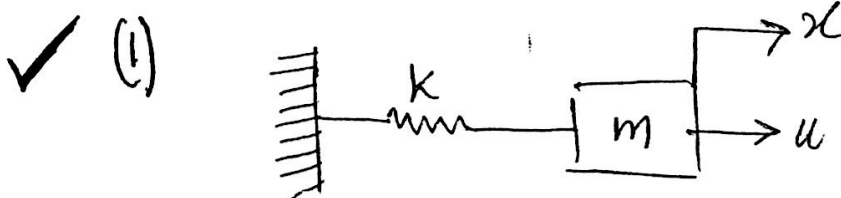
$$Z_1 = R_1 + \frac{1}{sC} \quad , \quad Z_2 = R_2$$

$$E_i(s) = \left(R_1 + \frac{1}{sC}\right) I(s)$$

$$E_o(s) = -R_2 I(s)$$

$$\frac{E_o(s)}{E_i(s)} = - \frac{R_2}{R_1 + \frac{1}{sC}} = - \frac{R_2 sC}{R_2 sC + 1}$$

Find the mathematical models of the mechanical systems shown below:-

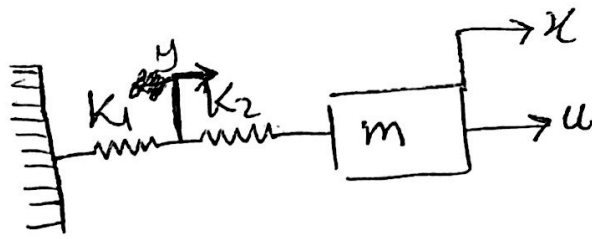


$$m\ddot{x} + kx = u$$

$$s^2 m x(s) + k x(s) = u(s)$$

$$x(s) (k + s^2 m) = u(s) \implies \frac{x(s)}{u(s)} = \frac{1}{k + s^2 m}$$

(2)



$$m\ddot{x} + k_2(x-y) = u$$

$$k_1 y = k_2(x-y)$$

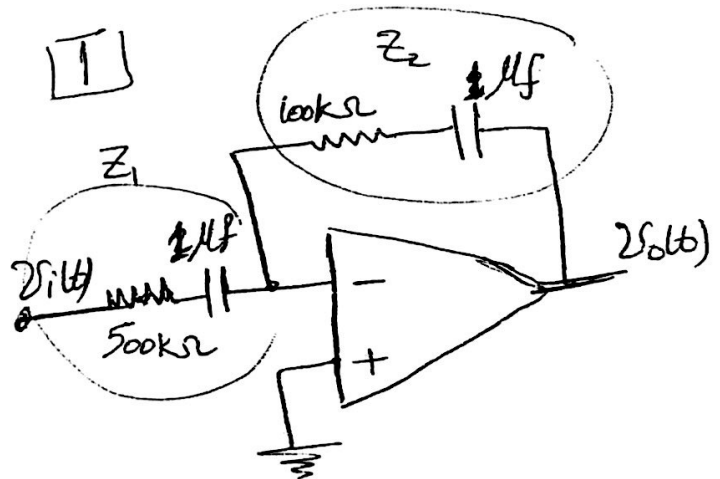
$$k_1 y + k_2 y = k_2 x$$

$$y = \frac{k_2}{k_1 + k_2} x$$

$$m\ddot{x} + \frac{k_1 k_2}{k_1 + k_2} x = u \Rightarrow \frac{x(s)}{U(s)} = \frac{1}{ms^2 + \frac{k_1 k_2}{k_1 + k_2}}$$

Find the transfer function $\frac{V_o(s)}{V_i(s)}$ for each op. amp. *

Circuit shown below:



$$Z(s) = 500 \times 10^3 + \frac{1}{s \times 10^{-6}}$$

$$Z_1(s) = 500 \times 10^3 + \frac{10^6}{s}$$

$$Z_2(s) = 100 \times 10^3 + \frac{1}{s \times 10^{-6}}$$

$$Z_2(s) = 100 \times 10^3 + \frac{10^6}{s}$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} \Rightarrow$$

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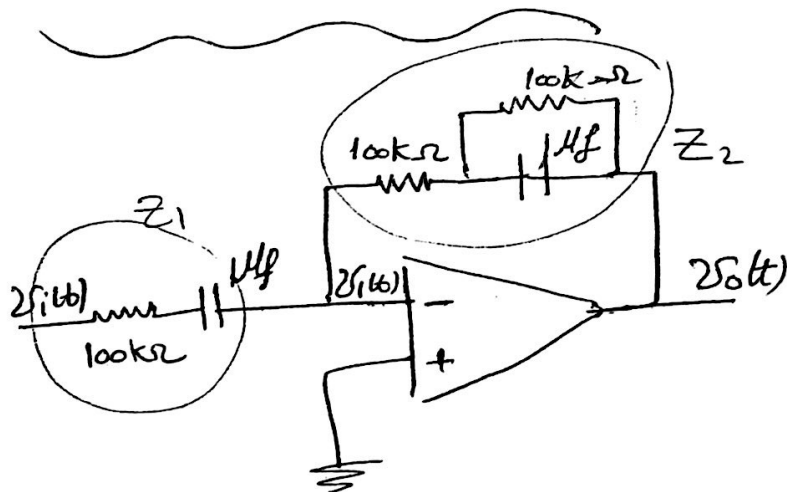
$$Z_1(s) = \frac{500 \times 10^3}{1} + \frac{10^6}{s} = \frac{500 \times 10^3 s + 10^6}{s}$$

$$Z_2(s) = 100 \times 10^3 + \frac{10^6}{s} = \frac{100 \times 10^3 s + 10^6}{s}$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2}{Z_1} = -\frac{\frac{100 \times 10^3 s + 10^6}{s}}{\frac{500 \times 10^3 s + 10^6}{s}} = -\frac{100 \times 10^3 s + 10^6}{500 \times 10^3 s + 10^6}$$

$$= -\frac{10^5 s + 10^6}{5 \times 10^5 s + 10^6} = -\frac{s + 10}{5s + 10} = \boxed{-\frac{1}{5} \left(\frac{s+10}{s+2} \right)}$$

2



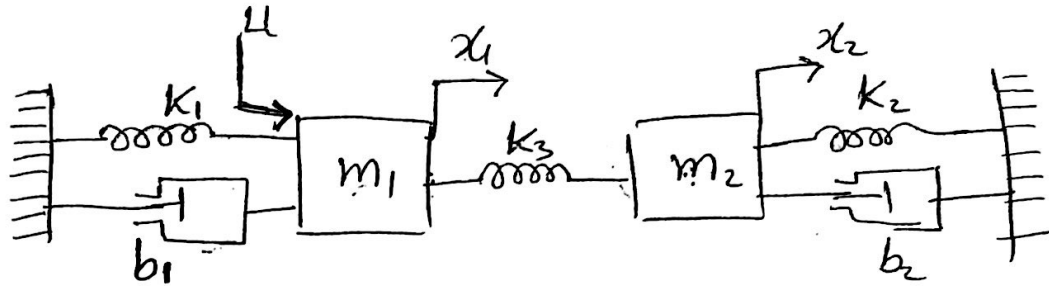
$$Z_1(s) = 10^5 + \frac{1}{10^{-6}} = \frac{10^5 s + 10^6}{s}$$

$$Z_2(s) = 10^5 + \frac{1}{10^{-6} s + \frac{1}{10^5}} = 10^5 \left(\frac{s+20}{s+10} \right)$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2}{Z_1} = \cancel{\left(\frac{s+20}{s+10} \right)} - \frac{(s+20)s}{(s+10)^2}$$

3

Find the transfer function $\frac{x_1(s)}{U(s)}$ and $\frac{x_2(s)}{U(s)}$ of the mechanical system shown below.



$$m_1 \ddot{x}_1 + b_1 \dot{x}_1 + k_1 x_1 + k_3 (x_1 - x_2) = U \quad \rightarrow [1]$$

$$m_2 \ddot{x}_2 + b_2 \dot{x}_2 + k_2 x_2 + k_3 (x_2 - x_1) = 0 \quad \rightarrow [2]$$

$$\cancel{m_1} \ddot{x}_1 + b_1 \dot{x}_1 + k_1 x_1 + k_3 x_1 = U + k_3 x_2 \quad \rightarrow [3]$$

$$m_2 \ddot{x}_2 + b_2 \dot{x}_2 + k_2 x_2 + k_3 x_2 = k_3 x_1 \quad \rightarrow [4]$$

$$\rightarrow (s^2 m_1 + s b_1 + k_1 + k_3) x_1(s) = U(s) + k_3 x_2(s) \quad \rightarrow \text{X}$$

$$\rightarrow (s^2 m_2 + s b_2 + k_2 + k_3) x_2(s) = k_3 x_1(s) \quad \rightarrow \text{X}$$

$$(s^2 m_1 + s b_1 + k_1 + k_3) x_1(s) = U(s) + k_3^2 x_1(s) \frac{1}{s^2 m_2 + s b_2 + k_2 + k_3}$$

$$\frac{x_1(s)}{U(s)} = \frac{s^2 m_2 + s b_2 + k_2 + k_3}{(s^2 m_1 + s b_1 + k_1 + k_3)(s^2 m_2 + s b_2 + k_2 + k_3) - k_3^2}$$

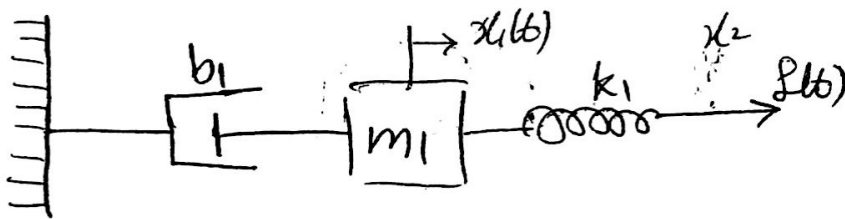
[4]

$$\frac{x_2(s)}{x_1(s)} = \frac{k_3}{s^2 m_2 + s b_2 + k_2 + k_3}$$

$$\frac{x_2(s)}{f(s)} = \frac{x_2(s)}{x_1(s)} \cdot \frac{x_1(s)}{f(s)} = \frac{k_3}{(s^2 m_1 + s b_1 + k_1 + k_3)(s^2 m_2 + s b_2 + k_2 + k_3)}$$

$\hookrightarrow -k_3^2$

Find the transfer function $G(s) = \frac{x_1(s)}{f(s)}$, for the translational mechanical system shown below.



$$m_1 \ddot{x}_1 + b_1 \dot{x}_1 + k_1 (x_1 - x_2) = 0$$

$$(s^2 m_1 + s b_1 + k_1) x_1(s) - k_1 x_2(s) = 0 \longrightarrow (1)$$

$$k_1 (x_2 - x_1) = f(t)$$

$$k_1 x_2(s) - k_1 x_1(s) = f(s) \longrightarrow (2)$$

$$x_2(s) = \frac{f(s) + k_1 x_1(s)}{k_1}$$

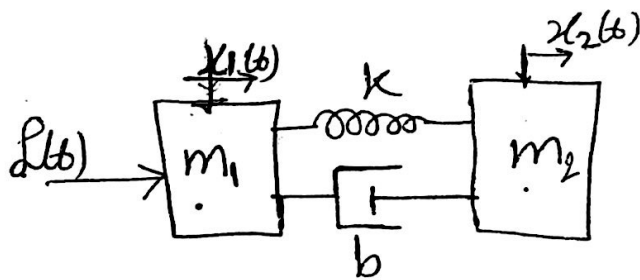


$$(s^2 m_1 + s b_1 + k_1) x_1(s) = k_2 \left(\frac{f(s) + k_1 x_1(s)}{k_1} \right)$$

$$k_1 (s^2 m_1 + s b_1 + k_1) x_1(s) - k_1 k_2 x_1(s) = k_2 f(s)$$

$$G(s) = \frac{x_1(s)}{f(s)} = \frac{k_2}{k_1 (s^2 m_1 + s b_1 + k_1) - k_1 k_2}$$

✓ Find the transfer function $G(s) = \frac{x_1(s)}{f(s)}$



$$m_1 \ddot{x}_1 + b(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2) = f(t)$$

$$(s^2 m_1 + s b + k) x_1(s) - (s b + k) x_2(s) = f(s) \rightarrow \textcircled{1}$$

$$m_2 \ddot{x}_2 + b(\dot{x}_2 - \dot{x}_1) + k(x_2 - x_1) = 0$$

$$(s^2 m_2 + s b + k) x_2(s) - (s b + k) x_1(s) = 0 \rightarrow \textcircled{2}$$

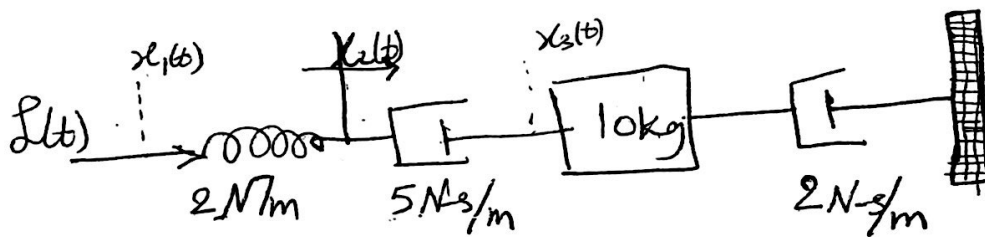
$\Downarrow$

$$\left. \begin{aligned} (s^2 m_1 + s b + k) x_1(s) - (s b + k) x_2(s) &= f(s) \\ -(s b + k) x_1(s) + (s^2 m_2 + s b + k) x_2(s) &= 0 \end{aligned} \right\}$$

$$X_2(s) = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} s^2 m_1 + b + k & f(s) \\ -(b+k) & 0 \end{bmatrix}}{\begin{bmatrix} s^2 m_1 + b + k & -(b+k) \\ -(b+k) & (s^2 m_2 + b + k) \end{bmatrix}} = A$$

$$G(s) = \frac{X_2(s)}{F(s)} = \frac{A}{f(s)}$$

✓ Find the transfer function, $G(s) = \frac{X_2(s)}{F(s)}$



$$2(x_1 - x_2) = f(t) \implies 2x_1(s) - 2x_2(s) = f(s) \longrightarrow \textcircled{1}$$

$$2(x_2 - x_1) + 5(\dot{x}_2 - \dot{x}_3) = 0$$

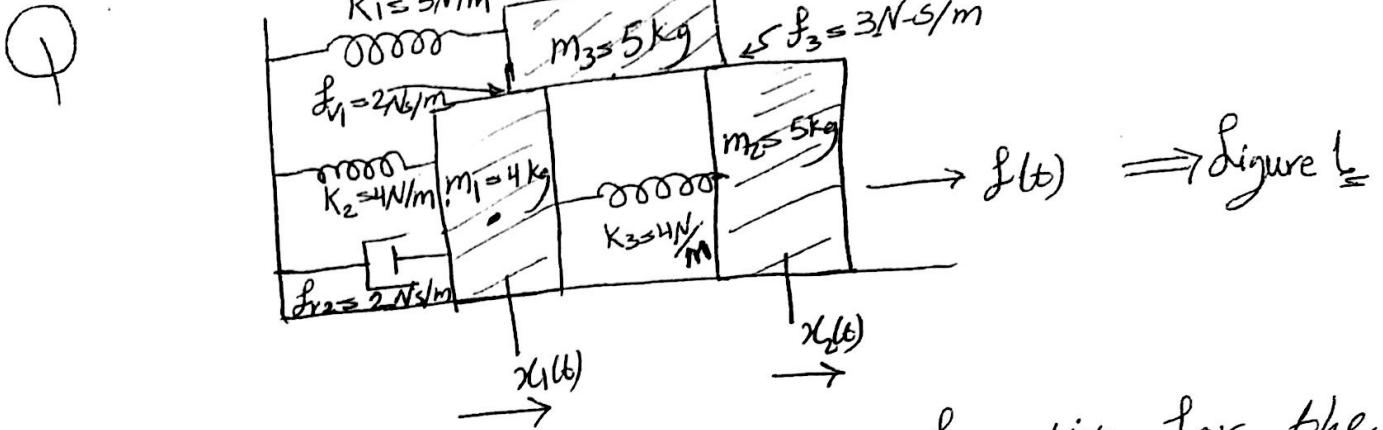
$$2x_2(s) - 2x_1(s) + 5s x_2(s) - 5s x_3(s) = 0 \longrightarrow \textcircled{2}$$

$$5(\ddot{x}_3 - \dot{x}_2) + 10\ddot{x}_3 + 2\dot{x}_3 = 0$$

$$5\dot{x}_3 - 5\dot{x}_2 + 10\ddot{x}_3 + 2\dot{x}_3 = 0$$

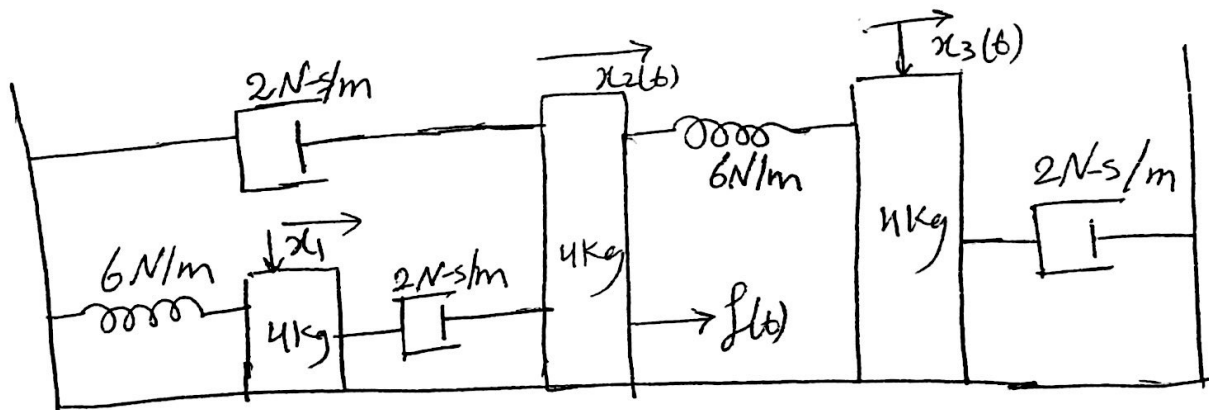
$$10\ddot{x}_3 + 7\dot{x}_3 - 5\dot{x}_2 = 0 \longrightarrow \textcircled{3}$$

$$10s^2 x_3 + 7s x_3 - 5s x_2 = 0 \longrightarrow \textcircled{3}$$



Write, but do not solve the equation of motion for the translational mechanical system shown in above Figure 1

Find the T.F $G(s) = \frac{x_3(s)}{F(s)}$, for the translational mechanical system shown in Figure 2



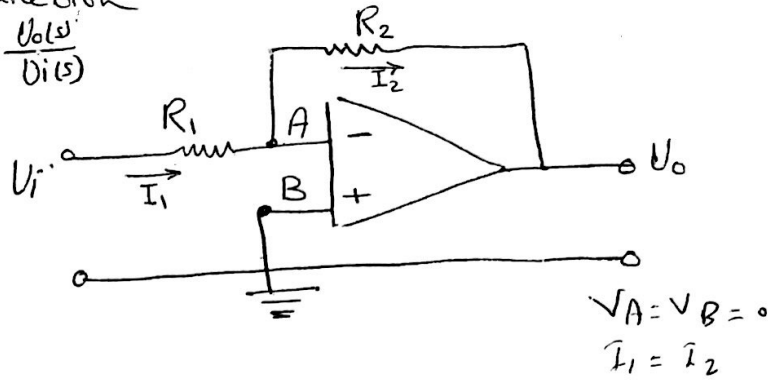
تمارين EE362

يوم الخميس 2014/4/3

م. أيمن أبو حسيبة

4/1/18

Transfer Function
of op. amp. $\frac{U_o(s)}{U_i(s)}$

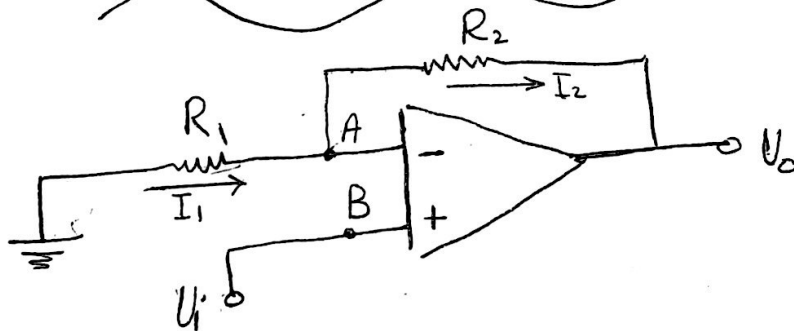


$$I_1 = \frac{U_i - U_A}{R_1}, \quad I_2 = \frac{U_A - U_o}{R_2}$$

$$I_1 = I_2, \quad U_A = U_B = 0$$

$$\frac{U_i - U_A}{R_1} = \frac{U_A - U_o}{R_2} \implies \frac{U_i}{R_1} = -\frac{U_o}{R_2} \implies \boxed{\frac{U_o}{U_i} = -\frac{R_2}{R_1}}$$

$$\boxed{\frac{U_o(s)}{U_i(s)} = -\frac{R_2}{R_1}}$$



$$I_1 = \frac{0 - U_A}{R_1} \implies I_1 = -\frac{U_A}{R_1} = -\frac{U_i}{R_1} \text{ from}$$

$$I_2 = \frac{U_A - U_o}{R_2} \implies I_2 = \frac{U_i - U_o}{R_2} \text{ from}$$

$$\boxed{U_A = U_B = U_i}$$

||

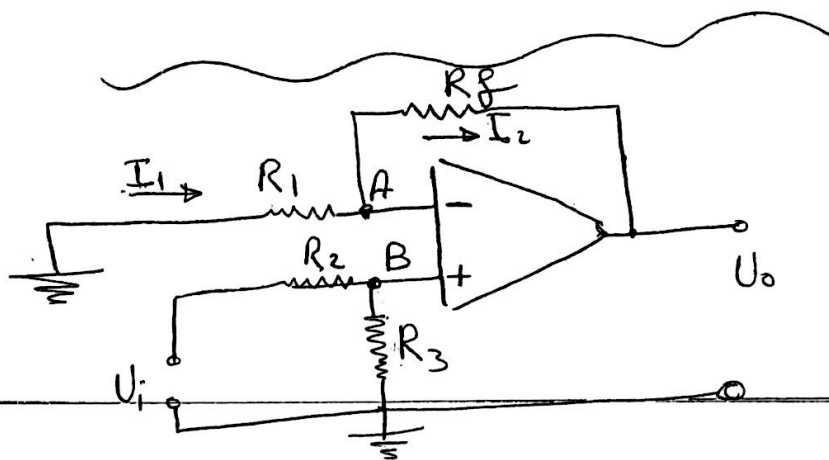
$$I_1 = I_2$$

$$\frac{-U_i}{R_1} = \frac{U_i - U_o}{R_2} \Rightarrow \frac{U_i}{R_2} + \frac{U_i}{R_1} = \frac{U_o}{R_2} \Rightarrow U_i \left(\frac{1}{R_2} + \frac{1}{R_1} \right) = \frac{U_o}{R_2}$$

a

$$\frac{U_o}{U_i} = R_2 \left(\frac{1}{R_2} + \frac{1}{R_1} \right) = \left(1 + \frac{R_2}{R_1} \right)$$

$$\boxed{\text{so } \frac{U_o(s)}{U_i(s)} = 1 + \frac{R_2}{R_1}} \quad \neq$$



$$I_1 = \frac{0 - U_A}{R_1}, \quad I_2 = \frac{U_A - U_o}{R_f}$$

$$U_A = U_B = \frac{U_i R_3}{R_2 + R_3}$$

$$I_1 = I_2$$

$$\frac{-U_A}{R_1} = \frac{U_A - U_o}{R_f} \Rightarrow \frac{U_o}{R_f} = \frac{U_A}{R_f} + \frac{U_A}{R_1}$$

[2]

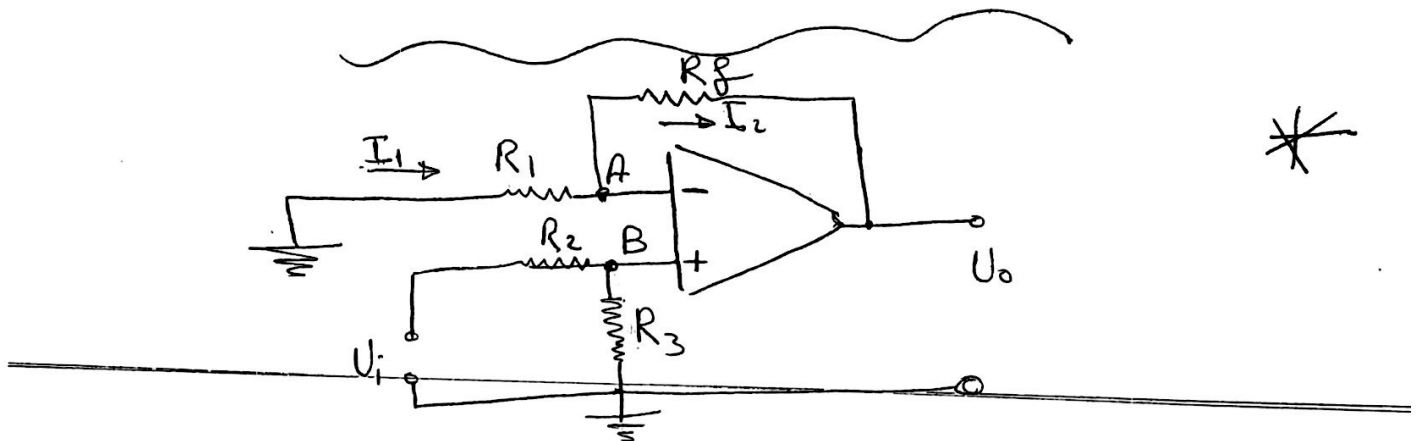
$$I_1 = I_2$$

$$\frac{-U_i}{R_1} = \frac{U_i - U_o}{R_2} \Rightarrow \frac{U_i}{R_2} + \frac{U_i}{R_1} = \frac{U_o}{R_2} \Rightarrow U_i \left(\frac{1}{R_2} + \frac{1}{R_1} \right) = \frac{U_o}{R_2}$$

$$\frac{U_o}{U_i} = R_2 \left(\frac{1}{R_2} + \frac{1}{R_1} \right) = \left(1 + \frac{R_2}{R_1} \right)$$

a

$$\boxed{\mathcal{L}_o \frac{U_o(s)}{U_i(s)} = 1 + \frac{R_2}{R_1}} \quad \neq$$



$$I_1 = \frac{0 - U_A}{R_1} \quad , \quad I_2 = \frac{U_A - U_o}{R_f}$$

$$U_A = U_B = \frac{U_i \cdot R_3}{R_2 + R_3}$$

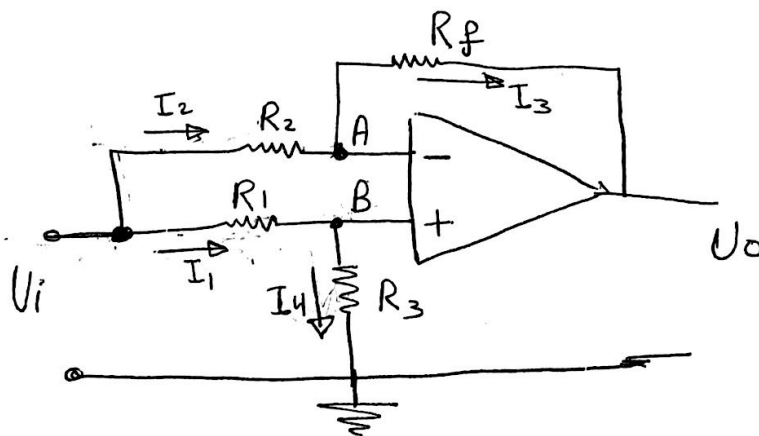
$$I_1 = I_2$$

$$\frac{-U_A}{R_1} = \frac{U_A - U_o}{R_f} \Rightarrow \frac{U_o}{R_f} = \frac{U_A}{R_f} + \frac{U_A}{R_1}$$

2

$$\frac{U_o}{R_f} = U_A \left(\frac{1}{R_f} + \frac{1}{R_1} \right) \Rightarrow \frac{U_o}{R_f} = \frac{U_i R_3}{R_2 + R_3} \left(\frac{1}{R_f} + \frac{1}{R_1} \right)$$

$$\boxed{\lim_{\omega \rightarrow \infty} \frac{U_o(s)}{U_i(s)} = \frac{R_3}{R_2 + R_3} \left(1 + \frac{R_f}{R_1} \right)}$$



$$I_2 = \frac{U_i - U_A}{R_2}, \quad I_3 = \frac{U_A - U_o}{R_f}$$

$$I_1 = \frac{U_i - U_B}{R_1}, \quad I_4 = \frac{U_B - 0}{R_3}$$

$$I_2 = I_3$$

$$\frac{U_i - U_A}{R_2} = \frac{U_A - U_o}{R_f}, \quad U_A = U_B = \frac{U_i R_3}{R_1 + R_3}$$

$$\frac{U_i}{R_2} - \frac{U_A}{R_2} = \frac{U_A}{R_f} - \frac{U_o}{R_f} \Rightarrow \frac{U_i}{R_2} + \frac{U_o}{R_f} = \frac{U_A}{R_f} + \frac{U_A}{R_2}$$

3

$$\frac{U_i}{R_2} + \frac{U_o}{R_f} = U_A \left(\frac{1}{R_f} + \frac{1}{R_2} \right)$$

$$\frac{U_i}{R_2} + \frac{U_o}{R_f} = U_i \frac{R_3}{R_1 + R_3} \left(\frac{1}{R_f} + \frac{1}{R_2} \right)$$

$$\frac{U_o}{R_f} = U_i \left[\frac{R_3}{R_1 + R_3} \left(\frac{1}{R_f} + \frac{1}{R_2} \right) - \frac{1}{R_2} \right]$$

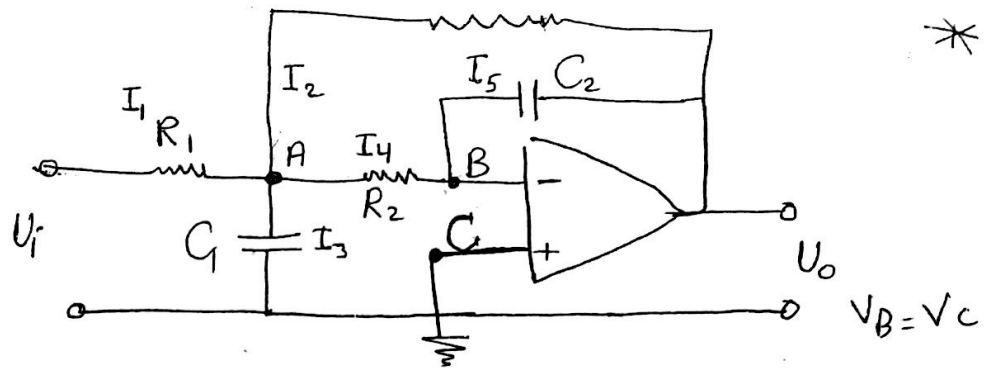
$$\frac{U_o}{U_i} = \frac{R_f R_3}{R_1 + R_3} \left(\frac{1}{R_f} + \frac{1}{R_2} \right) - \frac{R_f}{R_2}$$

$$= \frac{R_3}{R_1 + R_3} + \frac{R_f R_3}{R_2 (R_1 + R_3)} - \frac{R_f}{R_2}$$

$$= \frac{R_2 R_3 - R_f R_1}{R_2 (R_1 + R_3)}$$

$$\boxed{\lim_{s \rightarrow \infty} \frac{U_o(s)}{U_i(s)} = \frac{R_2 R_3 - R_f R_1}{R_2 (R_1 + R_3)}} \quad \neq$$

[4]



$$I_1 = \frac{U_i - U_A}{R_1}, \quad I_2 = \frac{U_A - U_o}{R_2}, \quad I_3 = C_1 \frac{dU_A}{dt} - \cancel{C_1 \frac{dU_o}{dt}}$$

$$I_4 = \frac{U_A - \cancel{U_B}}{R_2}, \quad I_5 = \cancel{C_2 \frac{dU_B}{dt}} - C_2 \frac{dU_o}{dt}$$

$$I_5 = -C_2 \frac{dU_o}{dt} \quad \text{because } U_B = U_C \\ U_B = U_C = 0$$

$$I_1 = I_2 + I_3 + I_4$$

$$I_4 = I_5 \implies \frac{U_A}{R_2} = -C_2 \frac{dU_o}{dt} \implies U_A = -R_2 C_2 \frac{dU_o}{dt}$$

$$\frac{U_i - U_A}{R_1} = \frac{U_A - U_o}{R_2} + C_1 \frac{dU_A}{dt} + \frac{U_A}{R_2}$$

$$\frac{U_i}{R_1} - \frac{U_A}{R_1} = \frac{U_A}{R_2} - \frac{U_o}{R_2} + C_1 \frac{dU_A}{dt} + \frac{U_A}{R_2}$$

$$\frac{U_i}{R_1} + \frac{U_o}{R_2} = \frac{U_A}{R_2} + \frac{U_A}{R_1} + C_1 \frac{dU_A}{dt} + \frac{U_A}{R_2}$$

[5]

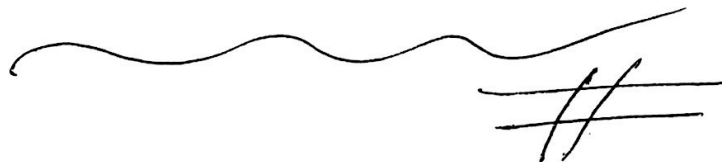
$$\frac{U_i}{R_1} + \frac{U_o}{R_3} = U_A \left(\frac{1}{R_3} + \frac{1}{R_1} + \frac{1}{R_2} \right) + C_1 \frac{dU_A}{dt} \quad \begin{matrix} \frac{d}{dt} = s \\ \frac{d^2}{dt^2} = s^2 \end{matrix}$$

$$\frac{U_i}{R_1} + \frac{U_o}{R_3} = -R_2 C_2 \frac{dU_o}{dt} \left(\frac{1}{R_3} + \frac{1}{R_1} + \frac{1}{R_2} \right) - C_1 C_2 R_2 \frac{d^2 U_o}{dt^2}$$

By taking the Laplace transform with zero initial condition we obtain —

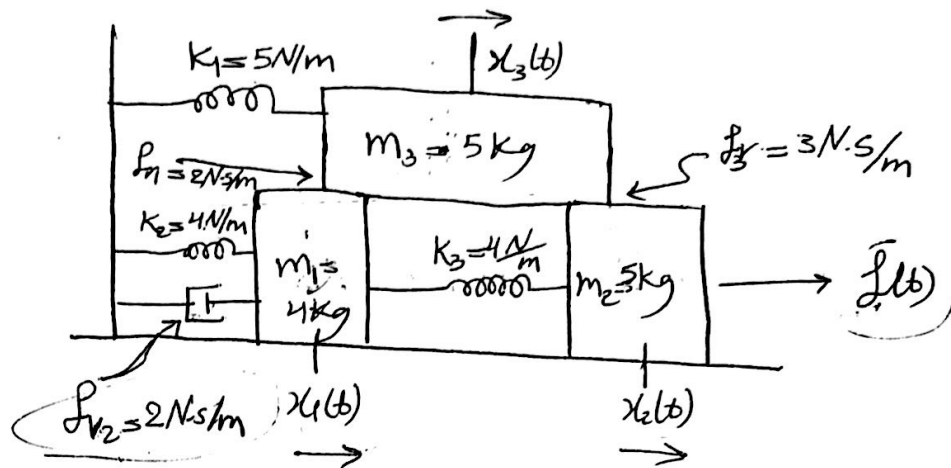
$$-C_1 C_2 R_2 s^2 U_o(s) - R_2 C_2 s U_o(s) \left(\frac{1}{R_3} + \frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{1}{R_3} U_o(s) = \frac{1}{R_1} U_i(s)$$

$$\frac{U_o(s)}{U_i(s)} = \frac{1}{-C_1 C_2 R_2 R_1 s^2 - R_2 C_2 s R_1 \left(\frac{1}{R_3} + \frac{1}{R_1} + \frac{1}{R_2} \right) - \frac{R_1}{R_3}}$$



6

① Write, but do not solve the equation of motion of the translational system shown in figure below.



$$m_1 \frac{d^2 x_1(t)}{dt^2} + K_2 x_1(t) + P_2 \frac{dx_1(t)}{dt} + P_1 \left(\frac{dx_1(t)}{dt} - \frac{dx_3(t)}{dt} \right) + K_3 (x_1(t) - x_2(t)) = 0$$

$$ms^2 X(s) - A' K X(s)$$

→ [1]

$$m_2 \frac{d^2 x_2(t)}{dt^2} + P_3 \left(\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt} \right) + K_3 (x_2(t) - x_1(t)) = f(t) \rightarrow [2]$$

$$m_3 \frac{d^2 x_3(t)}{dt^2} + K_1 x_3(t) + P_1 \left(\frac{dx_3(t)}{dt} - \frac{dx_1(t)}{dt} \right) + P_3 \left(\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt} \right) = 0$$

→ [3]

We taking the Laplace transform for the three last equations "assuming zero initial conditions"

For equation [1]

$$4s^2 \underline{x_1(s)} + 4 \underline{x_1(s)} + 2s \underline{x_1(s)} + 2s \underline{x_1(s)} - 2s \underline{x_3(s)} + 4 \underline{x_4(s)} - 4 \underline{x_2(s)} = 0$$

$$\boxed{(4s^2 + 4s + 8)x_1(s) - 4x_2(s) - 2sx_3(s) = 0} \rightarrow \boxed{1}$$

For equation [2]

$$5s^2 \underline{x_2(s)} + 3s \underline{x_2(s)} - 3s \underline{x_3(s)} + 4 \underline{x_2(s)} - 4 \underline{x_4(s)} = f(s)$$

$$\boxed{-4x_4(s) + (5s^2 + 3s + 4)x_2(s) - 3sx_3(s) = f(s)} \rightarrow \boxed{2}$$

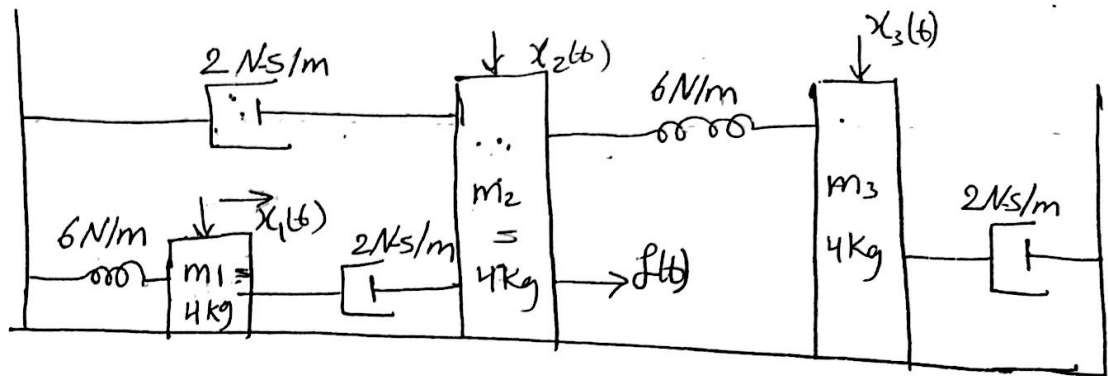
For equation [3]

$$5s^2 \underline{x_3(s)} + 5 \underline{x_3(s)} + 2s \underline{x_3(s)} - 2s \underline{x_4(s)} + 3s \underline{x_3(s)} - 3s \underline{x_2(s)} = 0$$

$$\boxed{-2sx_4(s) - 3sx_2(s) + (5s^2 + 5s + 5)x_3(s) = 0} \rightarrow \boxed{3}$$

~~///~~

find the T.F $G(s) = \frac{x_3(s)}{F(s)}$, for the translational mechanical system show in figure below



$$4 \frac{d^2 x_1(t)}{dt^2} + 6 x_1(t) + 2 \frac{dx_1(t)}{dt} - 2 \frac{dx_2(t)}{dt} = 0 \rightarrow [1]$$

$$4 \frac{d^2 x_2(t)}{dt^2} + 2 \frac{dx_2(t)}{dt} + 2 \frac{dx_2(t)}{dt} - 2 \frac{dx_1(t)}{dt} + 6 x_2(t) - 6 x_3(t) = f(t) \rightarrow [2]$$

$$4 \frac{d^2 x_3(t)}{dt^2} + 2 \frac{dx_3(t)}{dt} + 6 x_3(t) - 6 x_2(t) = 0 \rightarrow [3]$$

We taking the Laplace transform for the three last equations "assuming zero initial conditions"

for equation 1

$$4s^2 x_1(s) + 6 x_1(s) + 2s x_1(s) - 2s x_2(s) = 0$$

$$(4s^2 + 2s + 6) x_1(s) - 2s x_2(s) = 0 \rightarrow \boxed{1}$$

for equation 2

$$4s^2 x_2(s) + 2s x_2(s) + 2s x_2(s) - 2s x_1(s) + 6 x_2(s) - 6 x_3(s) = f(s)$$

$$(4s^2 + 4s + 6) x_2(s) - 2s x_1(s) - 6 x_3(s) = f(s) \rightarrow \boxed{2}$$

for equation 3

$$4s^2 x_3(s) + 2s x_3(s) + 6 x_3(s) - 6 x_2(s) = 0$$

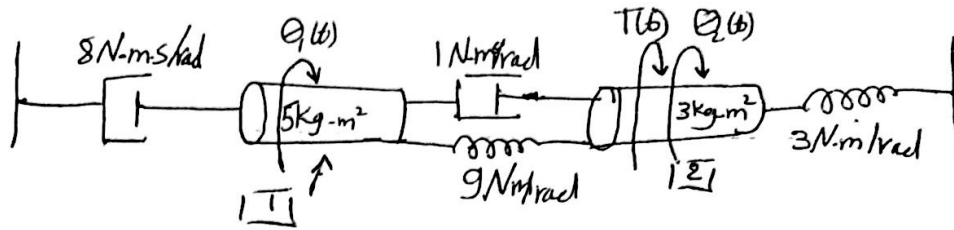
$$(4s^2 + 2s + 6) x_3(s) - 6 x_2(s) = 0 \rightarrow \boxed{3}$$

$$x_3(s) = \frac{\Delta_3}{\Delta} = \frac{\begin{vmatrix} 4s^2+2s+6 & -2s & 0 \\ -2s & 4s^2+4s+6 & f(s) \\ 0 & -6 & 0 \end{vmatrix}}{\begin{vmatrix} (4s^2+2s+6) & -2s & 0 \\ -2s & (4s^2+4s+6) & -6 \\ 0 & -6 & (4s^2+2s+6) \end{vmatrix}} = \frac{3f(s)}{s(8s^3+12s^2+26s+18)}$$

$$T.F \frac{x_3(s)}{f(s)} = \frac{3}{s(8s^3+12s^2+26s+18)} \quad \neq$$

Write, but do not solve, the equations of motion for the rotational mechanical system.

$$T(t) = f(t)$$



$$5 \frac{d^2 \theta_1(t)}{dt^2} + 8 \frac{d\theta_1(t)}{dt} + 1 \frac{d\theta_1(t)}{dt} - 1 \frac{d\theta_2(t)}{dt} + 9\theta_1(t) - 9\theta_2(t) = 0 \rightarrow [1]$$

$$3 \frac{d^2 \theta_2(t)}{dt^2} + 3\theta_2(t) + 1 \frac{d\theta_2(t)}{dt} - 1 \frac{d\theta_1(t)}{dt} + 9\theta_2(t) - 9\theta_1(t) = T(t) \rightarrow [2]$$

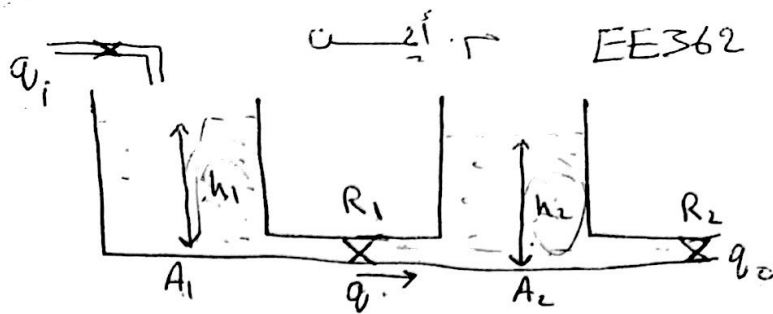
By taking the Laplace transform
we get

$$5s^2 \theta_1(s) + 8s \theta_1(s) + s \theta_1(s) - s \theta_2(s) + 9\theta_1(s) - 9\theta_2(s) = 0$$

$$\boxed{(5s^2 + 9s + 9) \theta_1(s) - (s + 9) \theta_2(s) = 0} \rightarrow [1]$$

$$3s^2 \theta_2(s) + 3\theta_2(s) + s \theta_2(s) - s \theta_1(s) + 9\theta_2(s) - 9\theta_1(s) = T(s)$$

$$\boxed{-(s + 9) \theta_1(s) + (3s^2 + s + 12) \theta_2(s) = T(s)} \rightarrow [2]$$

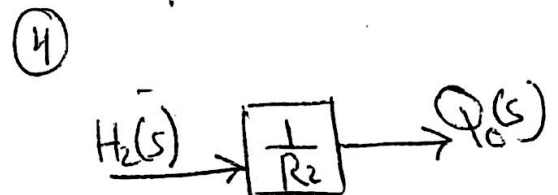
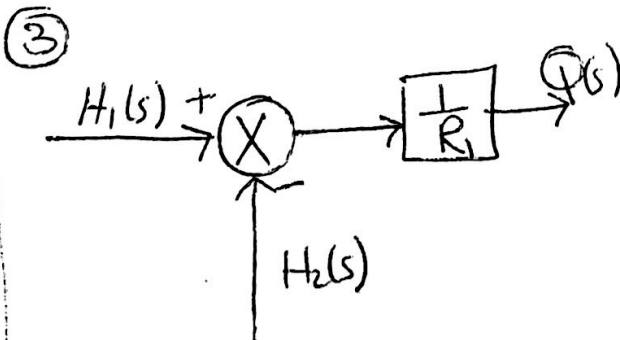
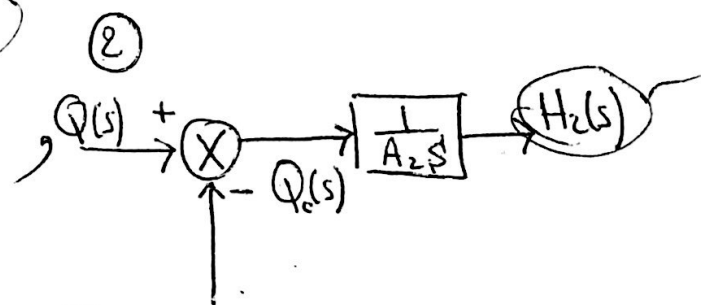
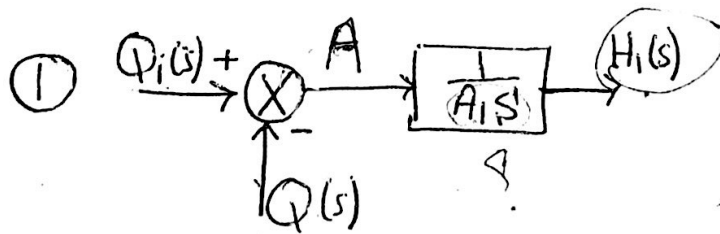


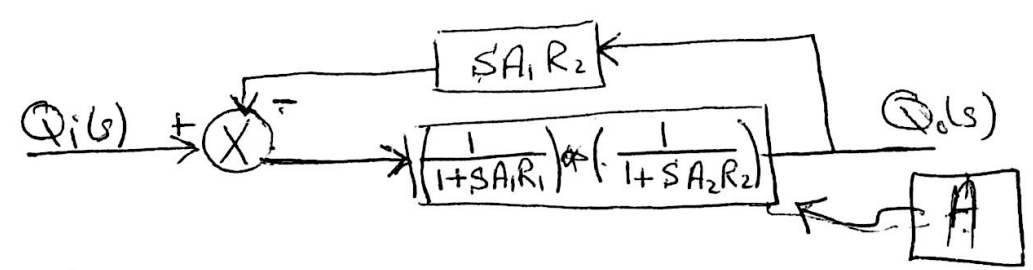
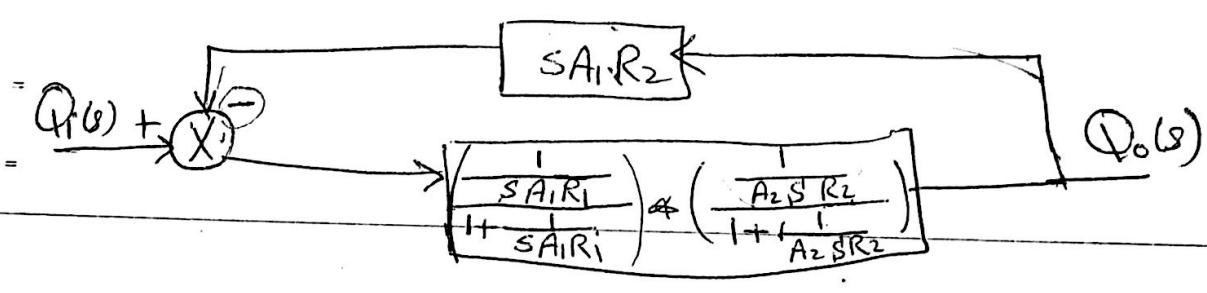
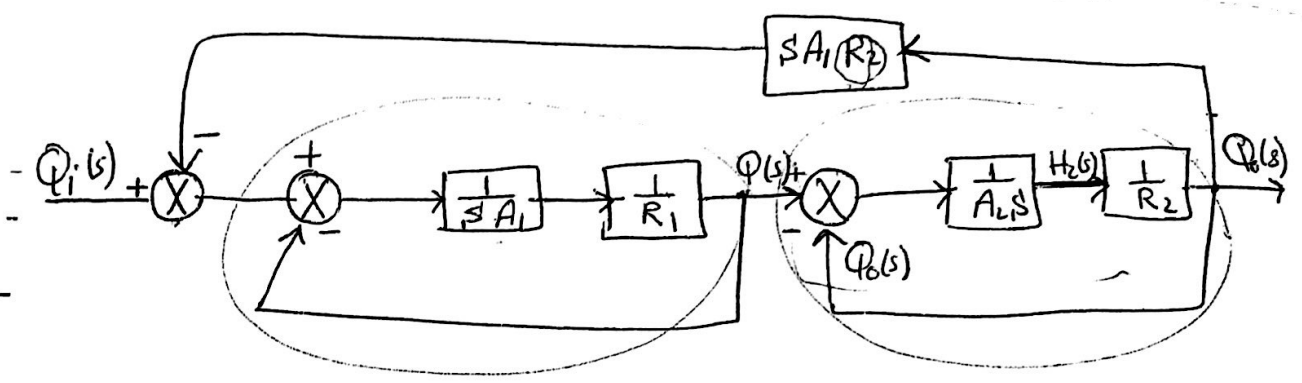
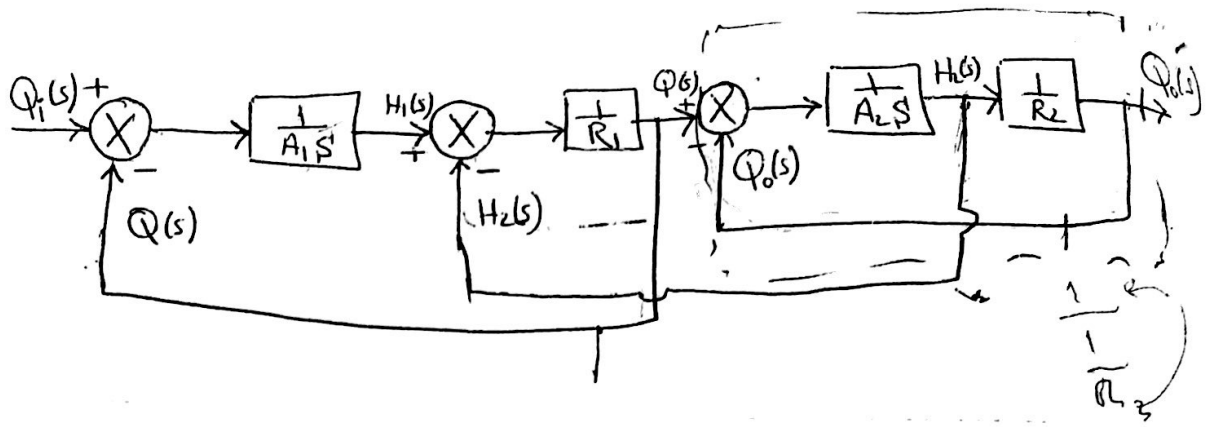
Find the block diagram for this system after that find the transfer function?

$$q_i - q = A_1 \frac{dh_1(t)}{dt}, \quad q - q_o = A_2 \frac{dh_2(t)}{dt}$$

$$q = \frac{h_1 - h_2}{R_1}, \quad q_o = \frac{h_2}{R_2}$$

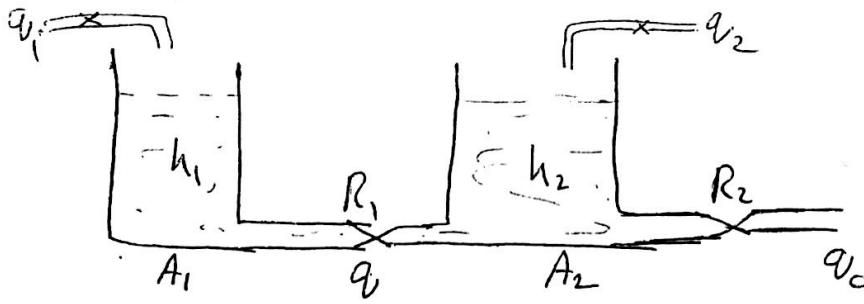
$$\begin{aligned} \textcircled{1} \quad & Q_i(s) - Q(s) = A_1 s H_1(s) \\ \textcircled{2} \quad & Q(s) - Q_o(s) = A_2 s H_2(s) \\ \textcircled{3} \quad & Q(s) = \frac{H_1(s) - H_2(s)}{R_1} \\ \textcircled{4} \quad & Q_o(s) = \frac{H_2(s)}{R_2} \end{aligned}$$





$$\frac{Q_o(s)}{Q_i(s)} = \frac{A}{1 + SA_1R_1 * A}$$

///



Find the block diagram for this system?

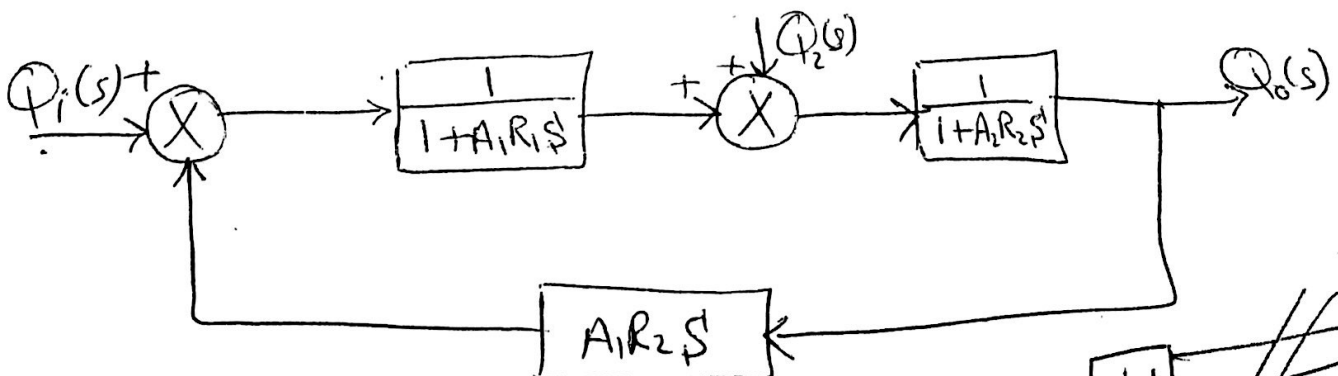
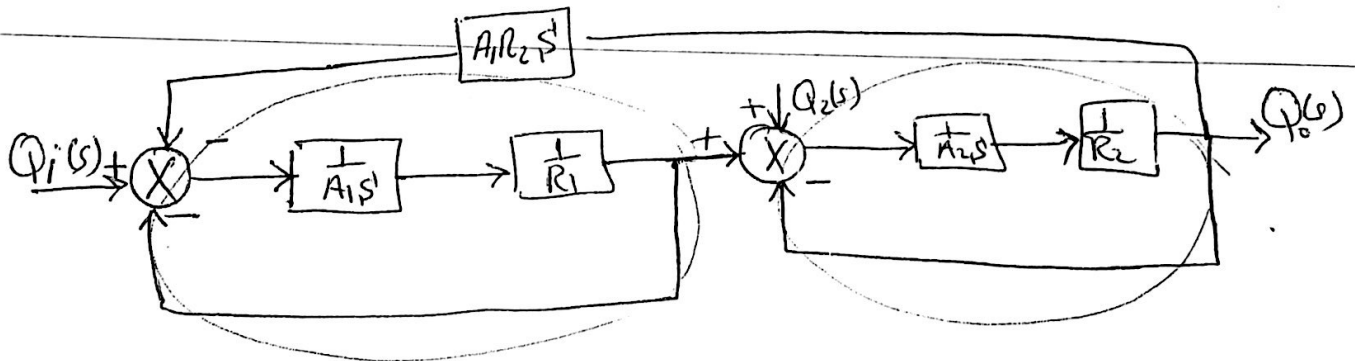
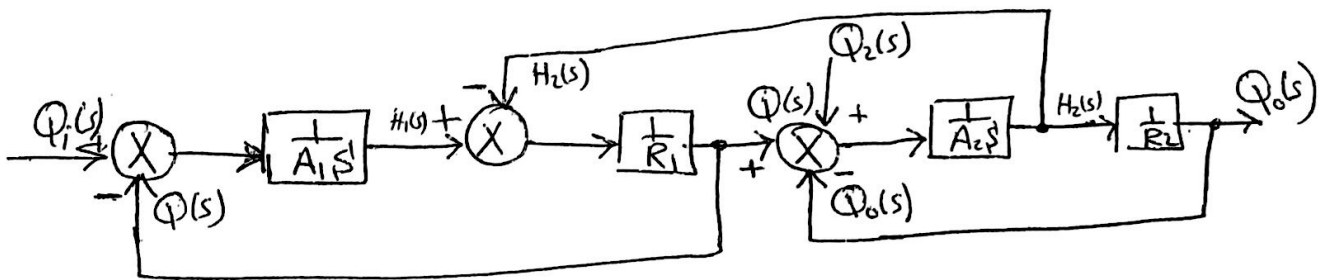
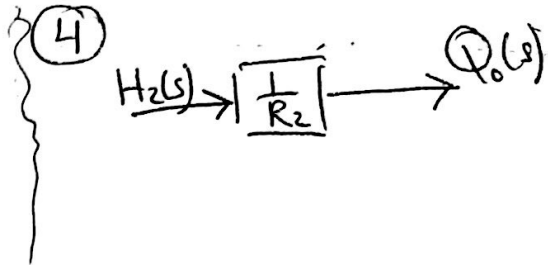
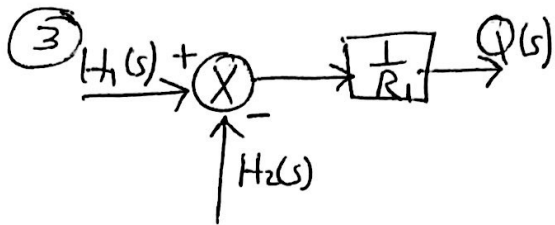
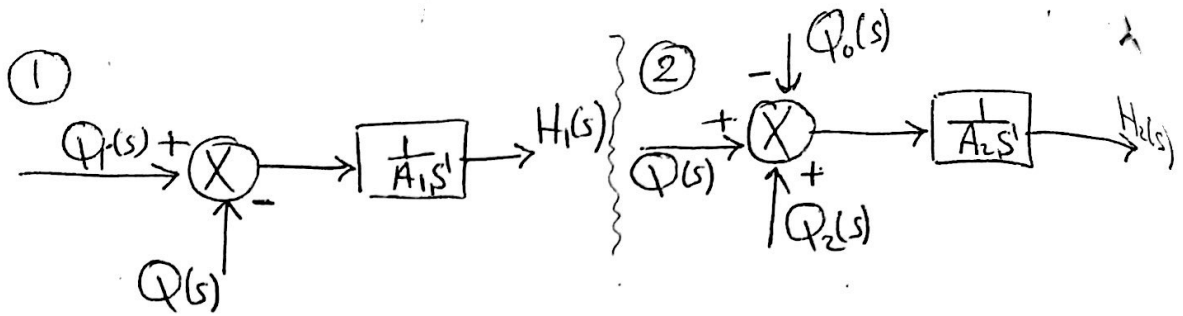
$$q_1 - q = A_1 \frac{dh_1}{dt}, \quad q + q_2 - q_0 = A_2 \frac{dh_2}{dt}$$

$$q = \frac{h_1 - h_2}{R_1}, \quad q_0 = \frac{h_2}{R_2}$$

$$\textcircled{1} \frac{Q_1(s) - Q(s)}{A_1 s} = H_1(s), \quad \textcircled{2} \frac{Q(s) + Q_2(s) - Q_0(s)}{A_2 s} = H_2(s)$$

$$\textcircled{3} Q(s) = \frac{H_1(s) - H_2(s)}{R_1}, \quad Q_0(s) = \frac{H_2(s)}{R_2}$$

3



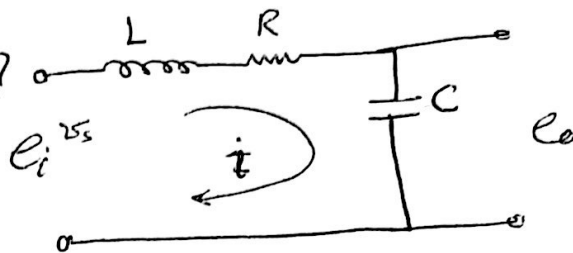
4 //

$$\begin{aligned} \dot{x} &= s f(s) - f(0) \\ \ddot{x} &= s^2 f(s) - s f(0) - \dot{f}(0) \end{aligned}$$

Find the

Transfer function?

$$\frac{2E(s)}{2S(s)}$$



$$L \frac{di}{dt} + Ri + \frac{1}{C} \int i dt = e_i \rightarrow \text{II}$$

$$\frac{1}{C} \int i dt = e_o$$

by taking L.T we get

$$Ls I(s) + R I(s) + \frac{1}{C} \cdot \frac{1}{s} \cdot I(s) = E_i(s) \rightarrow \text{①}$$

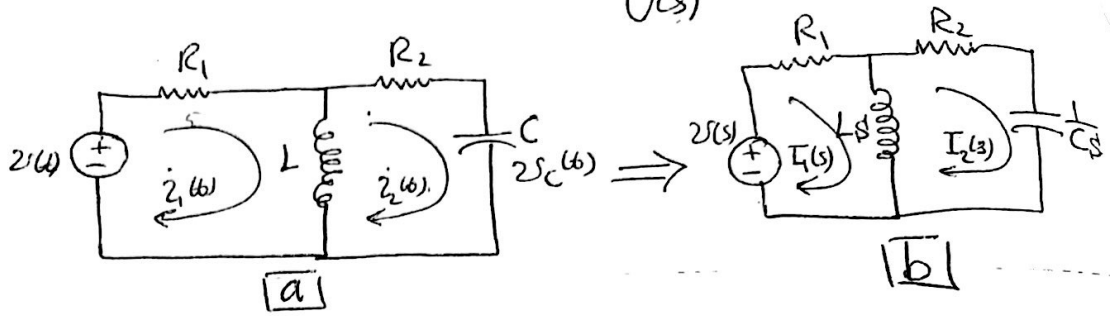
$$\frac{1}{C} \cdot \frac{1}{s} \cdot I(s) = E_o(s) \rightarrow \text{②}$$

$$I(s) \left[Ls + R + \frac{1}{Cs} \right] = E_i(s)$$

$$I(s) \left[\frac{1}{Cs} \right] = E_o(s)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{I(s) \left(\frac{1}{Cs} \right)}{I(s) \left(Ls + R + \frac{1}{Cs} \right)} \Rightarrow \frac{1}{LCs^2 + CRs + 1}$$

Find the transfer function $\frac{I_2(s)}{U(s)}$



$$R_1 I_1(s) + Ls I_1(s) - Ls I_2(s) = U(s) \quad \rightarrow ①$$

$$Ls I_2(s) + R_2 I_2(s) + \frac{1}{Cs} I_2(s) - Ls I_1(s) = 0 \quad \rightarrow ②$$

$$I_1(s) (R_1 + Ls) - (Ls) I_2(s) = U(s)$$

$$I_1(s) (-Ls) + (Ls + R_2 + \frac{1}{Cs}) I_2(s) = 0$$

$$\Delta = \begin{vmatrix} (R_1 + Ls) & (-Ls) \\ (-Ls) & (Ls + R_2 + \frac{1}{Cs}) \end{vmatrix} = (R_1 + Ls)(Ls + R_2 + \frac{1}{Cs}) - (L^2 s^2) = \checkmark$$

$$I_2 = \frac{\begin{vmatrix} (R_1 + Ls) & U(s) \\ (-Ls) & 0 \end{vmatrix}}{\Delta} = \frac{-Ls U(s)}{\Delta}$$

$$\text{Transfer Function} = \frac{I_2(s)}{U(s)} = \frac{-Ls}{\Delta} = \checkmark$$

[6]

EE362 تاريخ

6/5/2014

م. أيمن أبو حبيدة

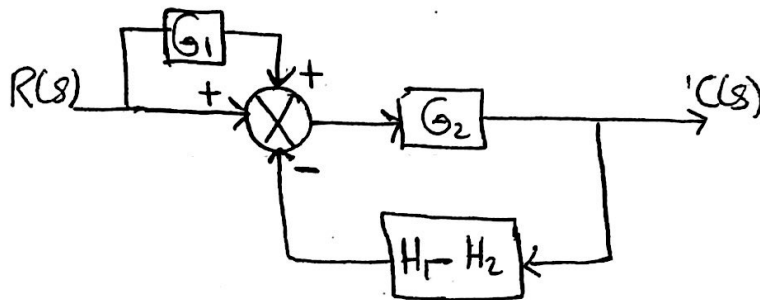
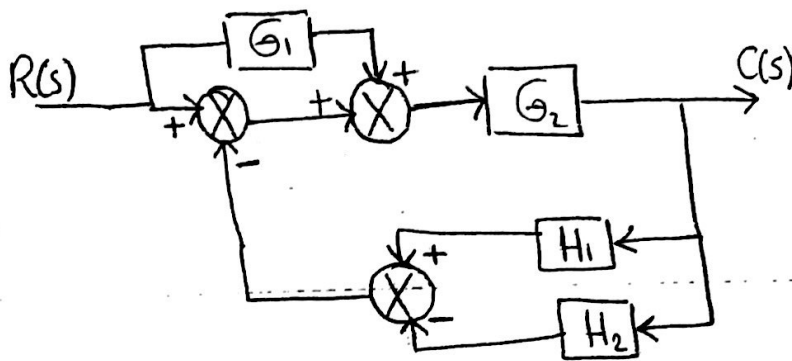
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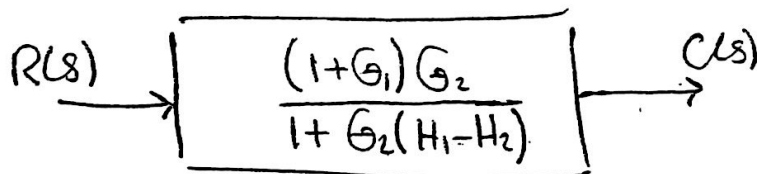
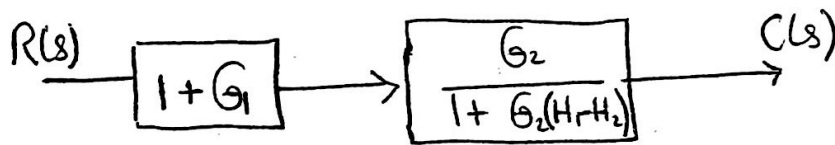
18/4/2014

Test I

Q2.a



as same as before

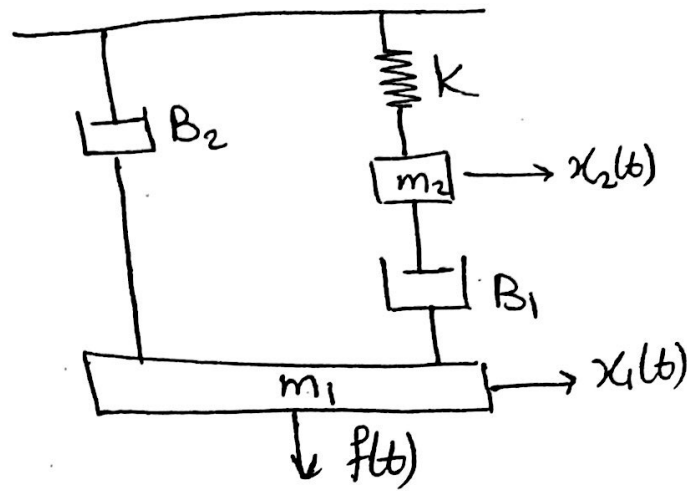


$$T.F = \frac{C(s)}{R(s)} = \frac{(1+G_1)G_2}{1+G_2(H_1-H_2)}$$

#

1

Q2. b



$$m_1 \frac{d^2 x_1(t)}{dt^2} + B_2 \frac{dx_1(t)}{dt} + B_1 \left(\frac{dx_1(t)}{dt} - \frac{dx_2(t)}{dt} \right) = f(t) \rightarrow [1]$$

$$m_2 \frac{d^2 x_2(t)}{dt^2} + K x_2(t) + B_1 \left(\frac{dx_2(t)}{dt} - \frac{dx_1(t)}{dt} \right) = 0 \rightarrow [2]$$

By taking Laplace transform

$$m_1 s^2 x_1(s) + B_2 s x_1(s) + B_1 s x_1(s) - B_1 s x_2(s) = f(s)$$

$$\boxed{(m_1 s^2 + B_2 s + B_1 s) x_1(s) - B_1 s x_2(s) = f(s)} \rightarrow [3]$$

$$m_2 s^2 x_2(s) + K x_2(s) + B_1 s x_2(s) - B_1 s x_1(s) = 0$$

$$\boxed{-B_1 s x_1(s) + (m_2 s^2 + B_1 s + K) x_2(s) = 0} \rightarrow [4]$$

$$\Delta = \begin{vmatrix} m_1 s^2 + B_2 s + B_1 s & -B_1 s \\ -B_1 s & m_2 s^2 + B_1 s + K \end{vmatrix} = A$$

$$\Delta_2 = \begin{vmatrix} m_1 s^2 + B_2 s + B_1 s & f(s) \\ -B_1 s & 0 \end{vmatrix} = B$$

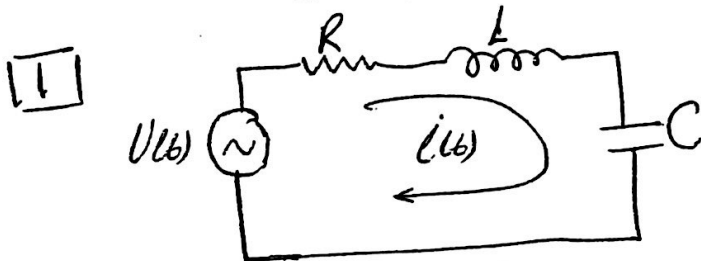
[2]

$$1 = [m_2 \dot{s}^2 + B_1 \dot{s} + K] (m_1 \dot{s}^2 + B_2 \dot{s} + B_1 \dot{s}) - (B_1^2 \dot{s}) = C = \Delta$$

$$B = [0 \times m_1 \dot{s}^2 + B_2 \dot{s} + B_1 \dot{s}] - [-B_1 \dot{s} f(s)] = B_1 \dot{s} f(s) = \Delta_2$$

$$x_2(s) = \frac{\Delta_2}{\Delta} = \frac{B_1 \dot{s} f(s)}{C} \Rightarrow \frac{x_2(s)}{f(s)} = \frac{B_1 \dot{s}}{C} \Rightarrow \text{T.F.}$$

Q3.a In this question, there are two answers and both of them are correct



$$iR + L \frac{di}{dt} + \frac{1}{C} \int i dt = U(t), \quad i = \frac{dq}{dt}$$

$$\frac{dq}{dt} R + L \frac{d^2 q}{dt^2} + \frac{q}{C} = U(t)$$

$$L \ddot{q} + R \dot{q} + \frac{q}{C} = U(t), \quad \text{output, } V_o = \frac{1}{C} \int i dt = \frac{q}{C}$$

$$x_1 = q \Rightarrow \boxed{\dot{x}_1 = \dot{q} = x_2} \rightarrow [1]$$

$$x_2 = \dot{q} \Rightarrow \boxed{\dot{x}_2 = \ddot{q} = -\frac{R}{L} x_2 - \frac{x_1}{LC} + \frac{U(t)}{L}} \rightarrow [2]$$

$$\boxed{\text{output } V_o = \frac{x_1}{C}} \rightarrow [3]$$

[3]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u$$

$$y = \begin{bmatrix} \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

~~///~~

another solution:-

$$V_L = L \frac{di_L}{dt}, \quad i_C = C \frac{dV_C}{dt}$$

$$x_1 = i_L \Rightarrow \dot{x}_1 = \frac{V_L}{L} =$$

$$x_2 = V_C \Rightarrow \dot{x}_2 = \frac{i_C}{C} =$$

$$V_i = V_R + V_L + V_C \Rightarrow V_L = V_i - V_R - V_C$$

$$V_L = U(t) - R i_L - V_C = U(t) - R x_1 - x_2$$

because $i = i_L = i_R = i_C$

$$\therefore \dot{x}_1 = \frac{V_L}{L} = \frac{U(t)}{L} - \frac{R}{L} x_1 - \frac{x_2}{L} \rightarrow [1]$$

$$\therefore \dot{x}_2 = \frac{i_C}{C} = \frac{i_L}{C} = \frac{x_1}{C} \rightarrow [2]$$

$$\text{output} \Rightarrow V_o = V_C = x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

~~///~~

[4]

Q3.b $T.F = \frac{1}{\Delta} \sum p_k \Delta_k$

$$p_1 = G_1 G_2 G_3 G_4, \quad p_2 = G_5 G_6 G_7 G_8$$

$$L_1 = G_2 H_2$$

$$L_2 = G_3 H_3$$

$$L_3 = G_6 H_6$$

$$L_4 = G_7 H_7$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_3 + L_1 L_4 + L_2 L_3 + L_2 L_4) - 0$$

$$\Delta_1 = 1 - (L_3 + L_4)$$

$$\Delta_2 = 1 - (L_1 + L_2)$$

$$T.F = \frac{1}{\Delta} (p_1 \Delta_1 + p_2 \Delta_2)$$

~~≠~~

Eng. Ayman

5

Specifications of the output response

If the step input is applied to the system [second order system for example], the output response will have the form shown in figure "1". The time specification parameters of this response can be defined as follows.

I) Rise time (t_r)

Is the time required for the system time response $y(t)$ to move from 10% and 90% of its final value $y(\infty)$.

and it can be approximated by

$$t_r \approx 1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1 \xrightarrow{\text{from (Norman S. Nise)}}$$

ζ - Damping ratio

ω_n - undamping natural frequency

II) Delay time (t_d)

Is the time required for the system time response $y(t)$ to reach 50% of its final value $y(\infty)$. And it can be expressed by

$$t_d = \frac{1.1 + 0.125\zeta + 0.469\zeta^2}{\omega_n} = \left| \frac{1 + 0.7\zeta}{\omega_n} \right|$$

III) Settling time (t_s)

Is the time required for the system time response $y(t)$ to enter and remain within a small band around the final value, typically, this band may $\pm 2\%$, $\pm 4\%$ even $\pm 5\%$ of

final value, where

$$\zeta = \frac{4}{\zeta \omega_n} \quad \text{for } \pm 2\%$$

$$\zeta = \frac{3}{\zeta \omega_n} \quad \text{for } \pm 5\%$$

1] peak time (t_p)

Is the time required for the system time response $y(t)$ to reach first peak of the overshoot

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

$\omega_d \Rightarrow$ Damped frequency

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

5] Percent overshoot

The percent overshoot (OS%) is defined as

$$OS\% = 100 \left[\frac{\text{maximum value of } y(t) - y(\infty)}{y(\infty)} \right]$$

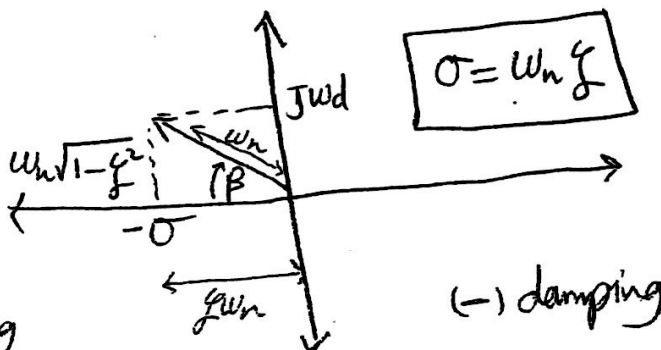
And it can be approximated by the following formula

$$OS\% = 100 e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}}$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

$$\beta = \tan^{-1} \frac{\omega_d}{\sigma}$$

(+) damping



$$\sigma = \omega_n \zeta$$

s_1, s_2 time constants of the system equal to $\frac{1}{\sigma}$

$$s = -\sigma \pm j\omega_d$$

$$= -\zeta \omega_n \pm j\omega_n \sqrt{1-\zeta^2}$$

$$\omega_n = \sqrt{\sigma^2 + \omega_d^2}$$

(-) damping

second order system

The general form of second order system can be written as

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

I] For $\zeta = 0$ (No damped system), the transient response is sinusoidal

II] For $\zeta = 1$ (Critical damped system), the transient response doesn't oscillate

III] For $0 < \zeta < 1$ (Under damped system), the transient response is oscillatory

IV] For $\zeta > 1$ (over-damped), the transient response doesn't oscillate

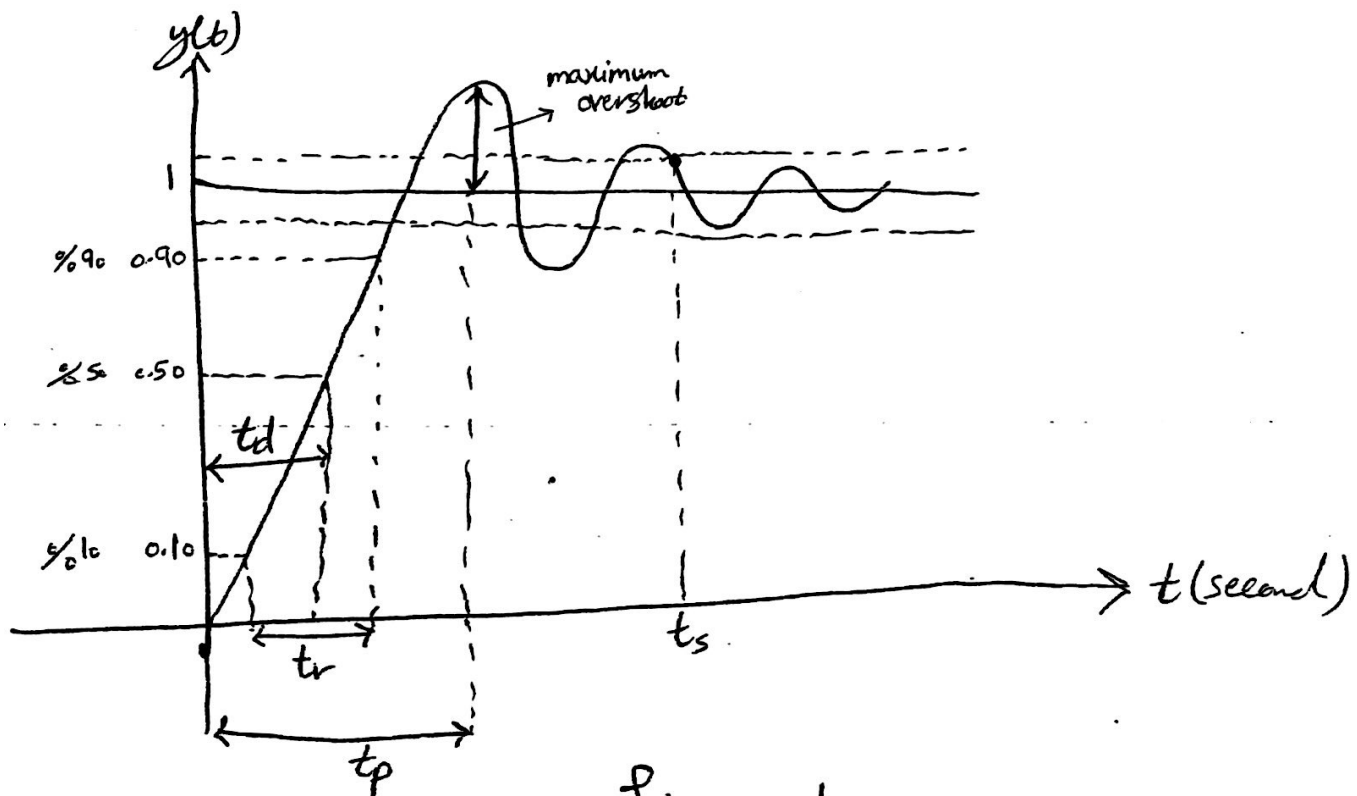
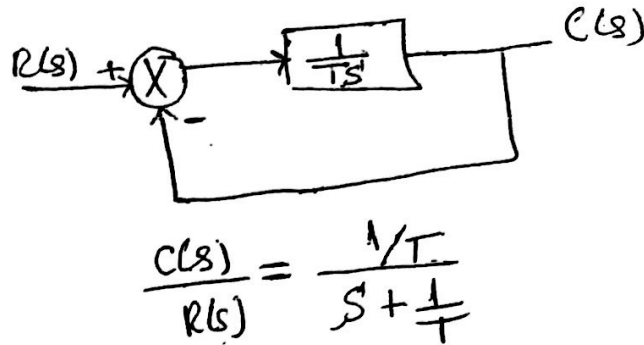


Figure 1

First order system

$$t_r = 2.2T \quad (\text{rise time})$$

$$t_s = 4T \quad (\text{settling time})$$



Q) For each of the second order systems that follow find ζ , ω_n , t_s , t_p , t_r and % OS

$$\text{a) } T(s) = \frac{16}{s^2 + 3s + 16}$$

$$\text{b) } T(s) = \frac{0.04}{s^2 + 0.02s + 0.04}$$

$$\text{c) } T(s) = \frac{1.05 \times 10^7}{s^2 + 1.6 \times 10^3 s + 1.05 \times 10^7}$$

For a) $\omega_n^2 = 16 \Rightarrow \boxed{\omega_n = 4}$, $2\zeta\omega_n = 3 \Rightarrow \boxed{\zeta = 0.375}$

$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.375 \times 4} = 2.667 \text{ second}, \quad t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}} = 0.8472 \text{ second}$$

$$\% \text{ OS} = 100 \times e^{\frac{-\zeta\pi}{\sqrt{1 - \zeta^2}}} = 85.45\%, \quad t_r = \frac{1}{\omega_n} (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1) = 0.356 \text{ second}$$

* try to solve b) and c)

Q For each pair of second order system specifications that follow, find the location of the second order pair of poles

1a) %OS = 12% , $t_s \leq 0.6$ second

1b) %OS = 10% , $t_p \leq 5$ second

1c) $t_s \leq 7$ second , $t_p \leq 3$ second

For 1a) $s = -\sigma \pm j\omega_d$

$$\sigma = \omega_n \zeta , \omega_d = \omega_n \sqrt{1 - \zeta^2} , \%OS = 100 e^{\frac{-\pi \zeta}{\sqrt{1 - \zeta^2}}}$$

$$0.12 = e^{\frac{-\pi \zeta}{\sqrt{1 - \zeta^2}}} \Rightarrow \ln 0.12 = \ln e^{\frac{-\pi \zeta}{\sqrt{1 - \zeta^2}}} \Rightarrow -2.1202 = \frac{-\pi \zeta}{\sqrt{1 - \zeta^2}}$$

$$\Rightarrow (-2.1202)^2 = \frac{(\pi \zeta)^2}{1 - \zeta^2} \Rightarrow 4.495 - 4.495 \zeta^2 = 9.8696 \zeta^2$$

$$14.3646 \zeta^2 = 4.495 \Rightarrow \zeta^2 = 0.31292 \Rightarrow \boxed{\zeta = 0.559}$$

$$t_s = \frac{4}{\zeta \omega_n} \Rightarrow 0.6 = \frac{4}{0.559 \omega_n} \Rightarrow \boxed{\omega_n \leq 11.926}$$

$$\sigma = \omega_n \zeta = 11.926 * 0.559 = 6.667$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 11.926 \sqrt{1 - (0.559)^2} = 9.88$$

b) Try to solve it by using these equations

$$\%OS = 100 e^{\frac{-\pi \zeta}{\sqrt{1-\zeta^2}}}, \quad t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

The answer $\boxed{s_1, s_2 = -0.4605 \pm j0.6287}$

For [C] $t_s = \frac{4}{\zeta \omega_n} \Rightarrow \gamma = \frac{4}{\zeta \omega_n}, \quad t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$

$\Downarrow \qquad \qquad \qquad \Downarrow$
 $\omega_n = \frac{4}{\gamma \zeta} \Rightarrow t_p = \frac{\pi}{\frac{4}{\gamma \zeta} \sqrt{1-\zeta^2}}$

$$(3)^2 = \frac{(\pi)^2}{\left(\frac{4}{\gamma \zeta}\right)^2 (1-\zeta^2)} \Rightarrow 9 = \frac{9.8696}{\frac{16-16\zeta^2}{49\zeta^2}} \Rightarrow$$

$$9 = \frac{483.61 \zeta^2}{16-16\zeta^2} \Rightarrow 483.61 \zeta^2 = 144 - 144 \zeta^2$$

$$627.61 \zeta^2 = 144 \Rightarrow \boxed{\zeta = 0.479}$$

$$\omega_n = \frac{4}{\gamma \zeta} = 1.929$$

$$\sigma = \omega_n \zeta = 0.571, \quad \omega_d = \omega_n \sqrt{1-\zeta^2} = 1.0471$$

$$\boxed{s_1, s_2 = -0.571 \pm j1.0471}$$

Q Find the transfer function of second order system that yields a 18.3% overshoot and a settling time of 1 second

$$OS\% = 100 e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}} \Rightarrow 18.3 = 100 e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}} \Rightarrow \ln\left(\frac{18.3}{100}\right) = \ln e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}}$$

$$\boxed{\zeta = 0.5549}$$

$$t_s = \frac{4}{\zeta \omega_n} \Rightarrow \omega_n = \frac{4}{0.5549} = 7.208$$

Find the percent overshoot, settling time and peak time for

$$T(s) = \frac{14.145}{(s^2 + 0.842s + 2.829)(s+5)}$$

$$(s+5) \rightarrow s = -5 \Rightarrow |R| = 5$$

$$s^2 + 0.842s + 2.829 \rightarrow s = \frac{-0.842 \pm \sqrt{0.842^2 - 4 \cdot 2.829}}{2} = \frac{-0.842 \pm j1.6286}{2} = -0.421 \pm j1.6286$$

$$\boxed{|R| = 1.67} \Rightarrow \text{It is a dominant pole.}$$

$$T(s) = \frac{14.145}{(s^2 + 0.842s + 2.829)(s+5)} = \frac{2.829}{s^2 + 0.842s + 2.829}$$

$$\omega_n^2 = 2.829 \Rightarrow \boxed{\omega_n = 1.6819}$$

$$2\zeta\omega_n = 0.842 \Rightarrow \boxed{\zeta = 0.25}$$

$$OS\% = 100 e^{-\frac{\pi \zeta}{\sqrt{1-\zeta^2}}} = 44.4\% \quad , \quad t_s = \frac{4}{\zeta \omega_n} = 9.5 \text{ second} \quad , \quad t_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = 1.93 \text{ second}$$